ABC on IBC

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We will introduce basic concepts of information-based complexity, shortly called IBC. The main emphasis will be on approximation of linear functionals defined on standard classes of d variate functions. The prime example will be multivariate integration. Such problems are approximated by algorithms that use finitely many functions values. The main question will be to find out the information complexity which is defined as the minimal number of function values needed to compute an ε approximation of the relative error between the exact and approximate solutions. We will explain why the information complexity is infinite in the worst case and in the average case error for Gaussian measures. It turns out that the information complexity is finite in the probabilistic setting but we must use nonlinear algorithm which are, however, easy to implement. Finally, we discuss different notions of tractability of such linear functionals in the probabilistic setting. We will see that sometimes such problems suffer the curse of dimensionality and discuss how to break the curse by introducing weights that measure the decaying importance of successive variables.