MCMC design-based non-parametric regression for rare-event. Application to nested risk computations.

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Résumé

We design and analyze an algorithm for estimating the mean of a function of a conditional expectation, when the outer expectation is related to a rare-event. The outer expectation is evaluated through the average along the path of an ergodic Markov chain generated by a Markov chain Monte Carlo sampler. The inner conditional expectation is computed as a non-parametric regression, using a least-squares method with a general function basis and a design given by the sampled Markov chain. We establish non asymp- totic bounds for the L2-empirical risks associated to this least-squares regression; this generalizes the error bounds usually obtained in the case of i.i.d. observations. Global error bounds are also derived for the nested expectation problem. Numerical results in the context of financial risk computations illustrate the performance of the algorithms.

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