Reproducing Kernels of Sobolev Spaces and Applications to Tractability

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The standard Sobolev space $W_2^s(R^d)$, with arbitrary positive integers s and d for which $s > d/2$, has the reproducing kernel

$$K_{d,s}(x,t) = \int_{R^d} \frac{\prod_{j=1}^d \cos\left(2\pi\,(x_j - t_j)u_j\right)}{1 + \sum_{0<|\alpha|_1\le s} \prod_{j=1}^d (2\pi\,u_j)^{2\alpha_j}}\,\mathrm{d}u$$

for all $x, t \in R^d$, where x_j, t_j, u_j, α_j are components of d-variate x, t, u, α, and $|\alpha|_1 = \sum_{j=1}^d \alpha_j$ with non-negative integers α_j; see Hegland and Marti (1986). We obtain a more explicit form for the reproducing kernel $K_{1,s}$ and find a closed form for the kernel $K_{d,\infty}$.

Knowing the form of $K_{d,s}$, we present applications on the best embedding constants of the Sobolev space $W_2^s(R^d)$ to $L_\infty(R^d)$, and on strong polynomial tractability of integration with an arbitrary probability density. We prove that the embedding constants are exponentially small in d, and the worst case integration errors of algorithms using n function values are also exponentially small in d.